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Prediction of shear strength of FRP-reinforced concrete beams without stirrups based on genetic programming

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ABSTRACT

The use of fibre reinforced polymer (FRP) bars to reinforce concrete structures has received a great deal of attention in recent years due to their excellent corrosion resistance, high tensile strength, and good non-magnetization properties. Due to the relatively low modulus of elasticity of FRP bars, concrete members reinforced longitudinally with FRP bars experience reduced shear strength compared to the shear strength of those reinforced with the same amounts of steel reinforcement. This paper presents a simple yet improved model to calculate the concrete shear strength of FRP-reinforced concrete slender beams ($a/d > 2.5$) without stirrups based on the gene expression programming (GEP) approach. The model produced by GEP is constructed directly from a set of experimental results available in the literature. The results of training, testing and validation sets of the model are compared with experimental results. All of the results show that GEP is a strong technique for the prediction of the shear capacity of FRP-reinforced concrete beams without stirrups. The performance of the GEP model is also compared to that of four commonly used shear design provisions for FRP-reinforced concrete beams. The proposed model produced by GEP provides the most accurate results in calculating the concrete shear strength of FRP-reinforced concrete beams among existing shear equations provided by current provisions. A parametric study is also carried out to evaluate the ability of the proposed GEP model and current shear design guidelines to quantitatively account for the effects of basic shear design parameters on the shear strength of FRP-reinforced concrete beams.

1. Introduction

In recent years, fibre reinforced polymer (FRP) bars have been adopted as a potential solution to the corrosion problems in concrete structures. In addition to their excellent non-corrosive characteristics, FRP reinforcements have high strength-to-weight ratio, good fatigue properties and electro-magnetic resistance [1,2]. There are fundamental differences between the steel and FRP reinforcements: the latter has a lower modulus of elasticity and linear stress–strain diagram up to rupture with no discernible yield point and different bond strength according to the type of FRP product. Due to the relatively low modulus of elasticity of FRP bars, concrete members reinforced longitudinally with FRP bars experience reduced shear strength compared to the shear strength of those reinforced with the same amounts of steel reinforcement. This fact is supported by the findings from the experimental investigations on FRP-reinforced concrete beams [3–5].

The applied shear stresses in a cracked reinforced concrete member without transverse reinforcement are resisted by various shear mechanisms. The Joint ASCE-ACI Committee 445 [6] assessed that the quantity of concrete shear strength V_c can be considered as a combination of five mechanisms activated after the formation of diagonal cracks: (1) shear stresses in uncracked compressed concrete; (2) aggregate interlock; (3) dowel action of the longitudinal reinforcing bars; (4) arch action; and (5) residual tensile stresses transmitted directly across the cracks. The contribution of the uncracked concrete in reinforced concrete members depends mainly on the concrete strength, f'_c, and on the depth of the uncracked zone, which is function of the longitudinal reinforcement properties. Aggregate interlock results from the resistance to relative slip between two rough interlocking surfaces of the crack, much like frictional resistance. The dowel action refers to the shear force resisting transverse displacement between two parts of a structural element split by a crack that is bridged by the reinforcement. Arching action occurs in deep members or in members in which the shear span-to-depth ratio (a/d) is less than 2.5. This is not a shear transfer mechanism in the sense that it does not transmit a tangential force to a nearby parallel plane, but permits the transfer of a vertical concentrated force to a reaction, thereby reducing the contribution of the other types of shear transfer. The basic explanation of residual tensile stresses is that when concrete first cracks, a clean break does not occur. The residual tension in cracked

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concrete has been found to be present for crack widths smaller than 0.15 mm [5,7].

Due to the relatively low modulus of elasticity of FRP composite material, concrete members reinforced with FRP bars will develop wider and deeper cracks than members reinforced with steel. Deeper cracks reduce the contribution to shear strength from the uncracked concrete due to the lower depth of concrete in compression. Wider cracks in turn reduce the contributions from aggregate interlock and residual tensile stresses. Additionally, due to the relatively small transverse strength of FRP bars and relatively wider cracks, the contribution of dowel action can be very small compared to that of steel. Finally, the overall shear capacity of concrete members reinforced with FRP bars as flexural reinforcement is lower than that of concrete members reinforced with steel bars [5].

Previous studies [4,8–10] concluded that current shear design guidelines are very conservative in calculating the shear capacity of FRP-reinforced concrete beams. Consequently, the excessive amount of FRP needed to resist shear could be both costly and likely to create reinforcement congestion problems [8]. Accordingly, the purpose of this paper is to develop a simple yet accurate model for predicting the shear strength of FRP-reinforced concrete slender beams ($a/d > 2.5$) without stirrups. GEP approach is also used to build empirical model. For building the model, shear capacity results of 104 specimens used in training, testing and validation sets for GEP model were obtained from the literature. Six main parameters that affect the shear strength of FRP-reinforced concrete members were selected for input variables. In the sets of the model, the concrete compressive strength (f'_c), beam width (b_w), effective depth (d), shear span-to-depth ratio (a/d), reinforcement ratio (ρ_f) and the ratio of modulus of elasticity of FPR to steel reinforcement (E_f/E_s) were entered as input variables, while shear strength value (V_{cf}) was used as output variable. The performance of the model was subsequently compared to results obtained from different shear design guidelines namely, the provisions of the American Concrete Institute (ACI) [11,12], the Canadian Standards Association (CSA) [13], the Japan Society of Civil Engineers (JSCE) [14], and The Canadian Network of Centres of Excellence on Intelligent Sensing for Innovative Structures (ISIS) [15]. A parametric study was also carried out to evaluate the ability of the proposed GEP model and current shear design guidelines to quantitatively account for the effects of basic shear design parameters on the shear strength of FRP-reinforced concrete beams.

2. Review of current design provisions

Due to the rapid increase of using FRP materials as reinforcement for concrete structures, there are international efforts to develop design guidelines. These efforts have resulted in the publishing of several codes and design guidelines. Most of the shear design provisions incorporated in these codes and guides on shear capacity of FRP-reinforced concrete beams have focused on modifying existing shear design equations for steel-reinforced concrete beams to account for the substantial differences between FRP and steel reinforcement. These provisions are generally based on the parallel truss model with 45° constant inclination diagonal shear cracks. This model identifies the shear strength of a reinforced concrete flexural member as the sum of the shear capacity of the concrete component V_{cf} and the shear reinforcement component V_s. In this paper, the concrete shear strength component V_{cf} of members longitudinally reinforced with FRP bars as recommended by ACI 440, ISIS Canada, CSA S806, and JSCE are reviewed and they are listed in Table 1. Note that all strength reduction factors used in the equations listed in the table for design purposes are set equal to one for comparison.

3. Genetic programming approach

Genetic programming (GP) is proposed by Koza [16]. It is a generalization of genetic algorithms (GAs) [17]. The most general form of a solution to a computer-modelled problem is a computer program. GP takes cognizance of this and attempts to use computer programs as its data representation. Similarly to GA, GP needs only the problem to be defined. Then, the program searches for a solution in a problem-independent manner [16–18].

GP breeds computer programs to solve problems by executing the following three steps:

(1) Generate an initial population of random compositions of the functions and terminals of the problem.
(2) Iteratively perform the following substeps until the termination criterion has been satisfied:
    (A) Execute each program in the population and assign it a fitness value using the fitness measure.

Table 1
Shear design equations for FRP-reinforced concrete beams without stirrups.

ACI 440-03	$V_{cf} = \frac{\rho_f E_f}{90\beta_1 f'_c} V_c \leqslant V_c$ V_c is calculated using ACI 318; $\beta_1 = 0.85 - 0.05\left(\frac{f_c\prime - 28}{7}\right) \geqslant 0.65$
ACI 440-06	$V_{cf} = \frac{2\sqrt{f'_c}}{5} b_w C \quad C = Kd$ $K = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f$ and $n_f = \frac{E_f}{E_c}$
CSA S806-02	$V_{cf} = 0.035 b_w d \left(f_c\prime \rho_f E_f \frac{V_f d}{M_f}\right)^{1/3}$ $0.1 b_w d\sqrt{f'_c} \leqslant V_{cf} \leqslant 0.2 b_w d\sqrt{f_c\prime}$ for $d \leq 300$ mm $V_{cf} = \frac{130}{1000+d} b_w d\sqrt{f_c\prime} \geq 0.08 b_w d\sqrt{f'_c}$ for $d > 300$ mm, and $\frac{V_f d}{M_f} \leqslant 1$
JSCE-97	$V_{cf} = \frac{\beta_d \beta_p \beta_n f_{vcd}}{\gamma_b} b_W d \quad \beta_p = 3\sqrt{\frac{100\rho_f E_f}{E_S}} \leq 1.5 \quad \beta_d = 4\sqrt{\frac{1000}{d}} \leq 1.5$ $f_{vcd} = 0.23\sqrt{f_{cd}} \leq 0.72$(MPa) γ_b and β_n are factors to account for strength reduction and axial force, respectively
ISIS Canada-01	$V_{cf} = 0.2 b_w d\sqrt{f_c\prime \frac{E_f}{E_s}}$ for $d \leqslant 300$ mm $V_{cf} = \frac{260}{1000+d} b_w d\sqrt{f_c\prime \frac{E_f}{E_s}} \geqslant 0.1 b_w d\sqrt{f_c\prime \frac{E_f}{E_s}}$ for $d > 300$ mm

Note: f'_c = compressive strength of concrete, b_w and d = beam's width and effective width, respectively, ρ_f = longitudinal reinforcement ratio; E_c, E_s and E_f = modulus of elasticity of concrete, steel and FRP longitudinal bars, respectively; M_f and V_f = moment and shear force at critical section, respectively.

(B) Create a new population of computer programs by applying the following operations. The operations are applied to computer program(s) chosen from the population with a probability based on fitness.
   (i) *Reproduction*: Copy an existing program to the new population.
   (ii) *Crossover*: Create new offspring program(s) for the new population by recombining randomly chosen parts of two existing programs, as seen in Fig. 1.
   (iii) *Mutation*. Create one new offspring program for the new population by randomly mutating a randomly chosen part of one existing program, as seen in Fig. 2.

(3) The program that is identified by the method of result designation (e.g., the best-so-far individual) is designated as the result of the genetic programming system for the run. This result may be a solution (or approximate solution) to the problem [19,20].

3.1. Gene expression programming approach

Ferreira [21] suggested a new algorithm based on GA and GP. This algorithm develops a computer program encoded in linear chromosomes of fixedlength. The GEP, which performs the symbolic regression using the most of the genetic operators of GA, fundamentally aims to find a mathematical function principal using a set of data presented [22,23].

The basic GEP algorithm is depicted in Fig. 3. To develop a GEP model, five components; the function set, terminal set, fitness function, control parameters and stop condition are required. After the problem is encoded for candidate solution and the fitness function is specified, the algorithm randomly creates an initial population of viable individuals (chromosomes) and then converts the each chromosome into an expression tree corresponding to a mathematical expression. Afterwards the predicted target is compared with the actual one and the fitness score for each

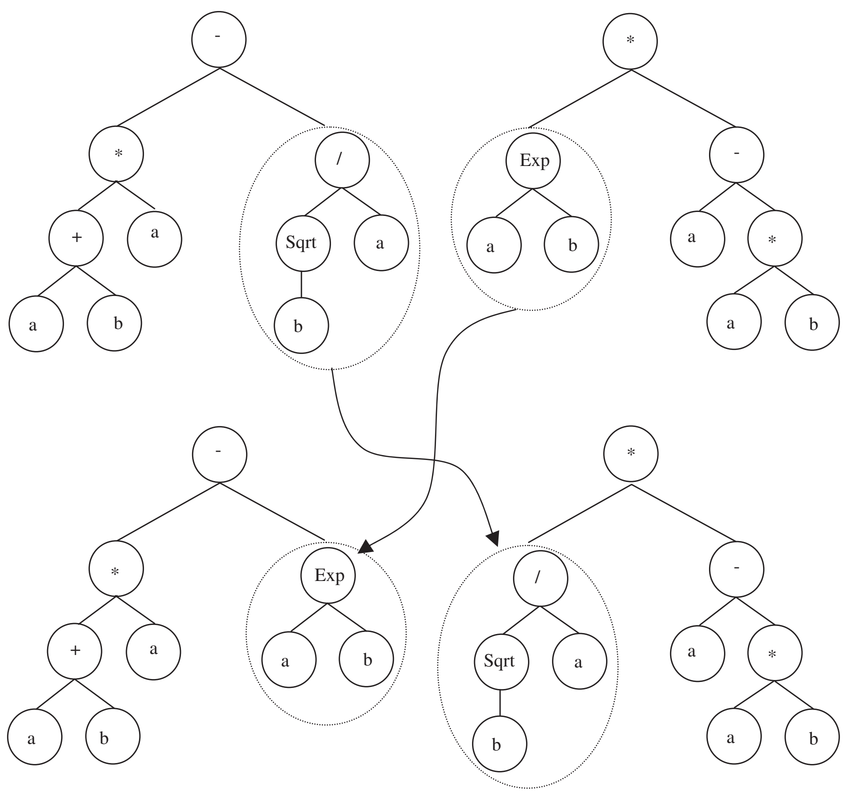


Fig. 1. Example of genetic programming crossover.

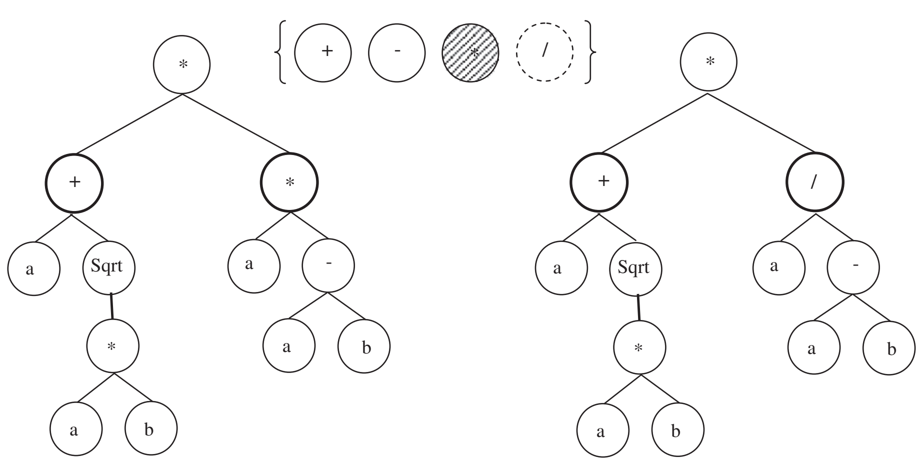


Fig. 2. Example of genetic programming mutation.

chromosome is determined. If it is sufficiently good, the algorithm stops. Otherwise, some of the chromosomes are selected using roulette wheel sampling and then mutated to obtain the new generations. This closed loop is continued until desired fitness score is achieved and then the chromosomes are decoded for the best solution of the problem [24,25].

GEP has two main elements such as the chromosomes and the expression trees (ETs). The chromosomes may be consisted of one or more genes which represents a mathematical expression. The mathematical code of a gene is expressed in two different languages called Karva Language [26,27]; such as the language of the genes and the language of the ETs. The genes have two main parts addressed as the head and the tail. The head includes some mathematical operators, variables and constants (+, -, $*$, /, $\sqrt{}$, sin, cos, 1, *a*, *b*, *c*) which are used to encode a mathematical expression. The tail just includes variables and constants (1, *a*, *b*, *c*) named as terminal symbols. Additional symbols are used if the terminal symbols in the head are inadequate to define a mathematical expression. A simple chromosome as linear string with one gene is encoded in Fig. 4. Its ET and the corresponding mathematical equation are also shown in same figure. The translation of ET to Karva Language is done by beginning to read from left to right in the top line of the tree and from top to bottom. The sequences of genes used in this method are similar to sequences of biological genes and have coding and non-coding parts. On the other hand more complex mathematical equations are defined by more than one chromosome referred to multigenic chromosomes. Joining of the genes is done by linking function such as addition, subtraction, multiplication, or division [23,25].

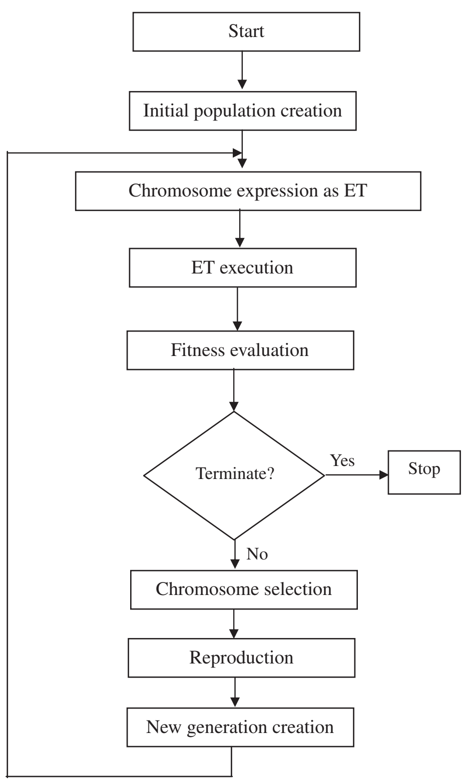


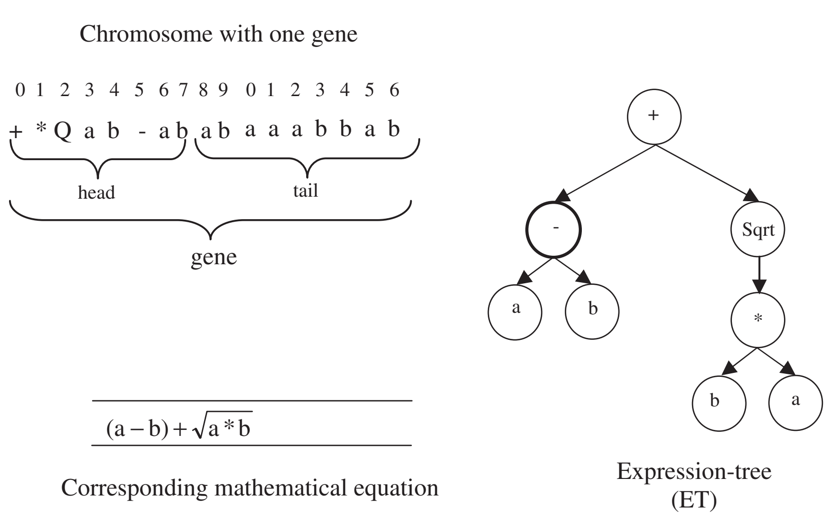
Fig. 3. The flowchart of gene expression programming [24].

3.2. Experimental database

In this study, shear strength results of 104 specimens given in Table 2 were collected from published literature [4,5,8,29–40]. The specimens included 91 beams and 13 one-way slabs; all were simply supported and were tested either in three-point or four-point bending. These specimens included two specimens reinforced with aramid FRP bars, 36 specimens reinforced with carbon FRP bars, and 66 specimens reinforced with glass FRP bars. All specimens had no transverse reinforcement and exhibited shear failure. The concrete compressive strength, f_c', of the test specimens ranged between 24.1 and 81.4 MPa. The reinforcement ratio, ρ_f, ranged between 0.25 and 3.02%; the shear span-to-depth ratio, a/d, ranged between 2.53 and 6.5; and the effective depth, *d*, ranged between 141 and 360 mm. Table 2 shows relevant details on the specimens.

3.3. Gene expression programming model

In the present study, six main parameters that affect the shear strength of FRP-reinforced concrete members without stirrups were selected for input variables. In training and testing of the GEP model, f_c', b_w, d, a/d, ρ_f and E_f/E_s were entered as input variables, while V_{cf} value was used as output variable. Among 104 experimental sets taken from the literature, 56 sets were randomly chosen as a training set for the GEP modeling and 28 sets were used as testing the generalization capacity of the proposed model.



$$(a-b)+\sqrt{a*b}$$

Fig. 4. Chromosome with one gene and its expression tree and corresponding mathematical equation.

Table 2
Training, testing and validation database for FRP-reinforced concrete members.

References	Beam	f'_c (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement		V_{exp} (kN)
						ρ_f (%)	E_f (GPa)	
[4]	1FRPa	36.3	229	225	914	1.11	40.3	39.1
[4]	1FRPb	36.3	229	225	914	1.11	40.3	38.5
[4]	1FRPc	36.3	229	225	914	1.11	40.3	36.8
[4]	2FRPa	36.3	178	225	914	1.42	40.3	28.1
[4]	2FRPb	36.3	178	225	914	1.42	40.3	35
[4]	2FRPc	36.3	178	225	914	1.42	40.3	32.1
[4]	3FRPa	36.3	229	225	914	1.66	40.3	40
[4]	3FRPb	36.3	229	225	914	1.66	40.3	48.6
[4]	3FRPc	36.3	229	225	914	1.66	40.3	44.7
[4]	4FRPa	36.3	279	225	914	1.81	40.3	43.8
[4]	4FRPb	36.3	279	225	914	1.81	40.3	45.9
[4]	4FRPc	36.3	279	225	914	1.81	40.3	46.1
[4]	5FRPa	36.3	254	224	914	2.05	40.3	37.7
[4]	5FRPb	36.3	254	224	914	2.05	40.3	51
[4]	5FRPc	36.3	254	224	914	2.05	40.3	46.6
[4]	6FRPa	36.3	229	224	914	2.27	40.3	43.5
[4]	6FRPb	36.3	229	224	914	2.27	40.3	41.8
[4]	6FRPc	36.3	229	224	914	2.27	40.3	41.3
[5]	S-C1	40	1000	165.3	1000	0.39	114	140
[5]	S-C2B	40	1000	165.3	1000	0.78	114	167
[5]	S-C3B	40	1000	160.5	1000	1.18	114	190
[5]	S-G1	40	1000	162.1	1000	0.86	40	113
[5]	S-G2	40	1000	159	1000	1.7	40	142
[5]	S-G2B	40	1000	162.1	1000	1.71	40	163
[5]	S-G3	40	1000	159	1000	2.44	40	163
[5]	S-G3B	40	1000	154.1	1000	2.63	40	168
[5]	CN-1	50	250	326	1000	0.87	128	77.5
[5]	GN-1	50	250	326	1000	0.87	39	70.5
[5]	CN-2	44.6	250	326	1000	1.24	134	104
[5]	GN-2	44.6	250	326	1000	1.22	42	60
[5]	CN-3	43.6	250	326	1000	1.72	134	124.5
[5]	GN-3	43.6	250	326	1000	1.71	42	77.5
[8]	BR1	40.5	200	225	600	0.25	145	36.1
[8]	BR2	49	200	225	600	0.5	145	47
[8]	BR3	40.5	200	225	600	0.63	145	47.2
[8]	BR4	40.5	200	225	600	0.88	145	42.7
[8]	BA3	40.5	200	225	800	0.5	145	49.7
[8]	BA4	40.5	200	225	950	0.5	145	38.5
[29]	Beam1	28.9	150	167.5	666.67	0.45	38	12.5
[29]	Beam3	28.9	150	212.3	666.67	0.71	32	17.5
[29]	Beam5	28.9	150	263	666.67	0.86	32	25.0
[29]	Beam7	50.15	150	162.6	666.67	1.39	32	17.5
[29]	Beam9	50.15	150	213.3	666.67	1.06	32	27.5
[29]	Beam11	50.15	150	262.12	666.67	1.15	32	30
[30]	CH-1.7	63	250	326	1000	1.71	135	130
[30]	GH-1.7	63	250	326	1000	1.71	42	87
[30]	CH-2.2	63	250	326	1000	2.2	135	174
[30]	GH-2.2	63	250	326	1000	2.2	42	115.5
[31]	8-2a	60.3	127	143	910	0.33	139	14.3
[31]	8-2b	60.3	127	143	910	0.33	139	12.9
[31]	8-2c	60.3	127	143	910	0.33	139	14.7
[31]	8-3a	61.8	159	141	910	0.58	139	19.8
[31]	8-3b	61.8	159	141	910	0.58	139	23.1
[31]	8-3c	61.8	159	141	910	0.58	139	17
[31]	11-2a	81.4	89	143	910	0.47	139	8.8
[31]	11-2b	81.4	89	143	910	0.47	139	11.7
[31]	11-2c	81.4	89	143	910	0.47	139	8.9
[31]	11-3a	81.4	121	141	910	0.76	139	14.3
[31]	11-3b	81.4	121	141	910	0.76	139	15.3
[31]	11-3c	81.4	121	141	910	0.76	139	16.6
[32]	1a-26	79.6	203	225	914	1.25	40.3	41.6
[32]	1b-26	79.6	203	225	914	1.25	40.3	30.4
[32]	1c-26	79.6	203	225	914	1.25	40.3	42.1
[32]	2a-26	79.6	152	225	914	1.66	40.3	31
[32]	2b-26	79.6	152	225	914	1.66	40.3	33.1
[32]	2c-26	79.6	152	225	914	1.66	40.3	33.5
[32]	3a-27	79.6	165	224	914	2.10	40.3	38.4
[32]	3b-27	79.6	165	224	914	2.10	40.3	32.2
[32]	3c-27	79.6	165	224	914	2.10	40.3	36.7
[32]	4a-37	79.6	203	224	914	2.56	40.3	48.3
[32]	4b-37	79.6	203	224	914	2.56	40.3	45.7
[32]	4c-37	79.6	203	224	914	2.56	40.3	45.2
[33]	G07N1	37.3	160	346	951.5	0.72	42	54.5

(*continued on next page*)

Table 2 (*continued*)

References	Beam	f_c' (MPa)	b_w (mm)	d (mm)	a (mm)	Reinforcement		V_{exp} (kN)
						ρ_f (%)	E_f (GPa)	
[33]	G07N2	37.3	160	346	951.5	0.72	42	63.7
[33]	G10N1	43.2	160	346	1149	1.1	42	42.7
[33]	G10N2	43.2	160	346	1149	1.1	42	45.5
[33]	G15N1	34.1	160	325	1150.5	1.54	42	48.7
[33]	G15N2	34.1	160	325	1150.5	1.54	42	44.9
[33]	C07N1	37.3	130	310	949	0.72	120	49.2
[33]	C07N2	37.3	130	310	949	0.72	120	45.8
[33]	C10N1	43.2	130	310	1150	1.1	120	47.6
[33]	C10N2	43.2	130	310	1150	1.1	120	52.7
[33]	C15N1	34.1	130	310	1150	1.54	120	55.9
[33]	C15N2	34.1	130	310	1150	1.54	120	58.3
[34]	V-G1-1	39.7	457	360	1219.2	0.96	40.5	108.1
[34]	V-G2-1	39.9	457	360	1219.2	0.96	37.6	94.7
[34]	V-A-1	40.3	457	360	1219.2	0.96	47.1	114.8
[34]	V-G1-2	42.3	457	360	1219.2	1.92	40.5	137
[34]	V-G2-2	42.5	457	360	1219.2	1.92	37.6	152.6
[34]	V-A-2	42.6	457	360	1219.2	1.92	47.1	177
[35]	BM7	24.1	178	279	750	2.3	40	53.4
[35]	BM8	24.1	178	287	750	0.77	40	36.1
[35]	BM9	24.1	178	287	750	1.34	40	40.1
[36]	GFRP1	28.6	305	157.5	710	0.73	40	26.8
[36]	GFRP2	30.1	305	157.5	913	0.73	40	28.3
[36]	GFP3	27	305	157.5	913	0.73	40	29.2
[36]	Hybrid1	28.2	305	157.5	913	0.73	40	28.5
[36]	Hybrid2	30.8	305	157.5	913	0.73	40	27.6
[37]	No.10	34.7	200	260	700	1.3	130	62.2
[38]	GB6	32.9	150	210	766.5	1.36	130	62.2
[39]	F-6-GF	39	154	222	700	1.55	34	19.5
[40]	No.1	34.3	150	250	750	1.51	105	45
[40]	No.6	34.3	150	250	750	3.02	105	46
[40]	No.15	34.3	150	250	750	2.27	105	40.5

20 sets of experimental data taken from the literature were also used as a validation set for the GEP. It should be noted that the validation set has not already been utilized in training and testing sets of the corresponding model. The limit values of input and output variables used in the GEP model are listed in Table 3.

For this problem, the fitness, f_i, of an individual program, i, is measured by

$$f_i = \sum_{j=1}^{C_t} (M - |C_{i,j)-T_j}|) \quad (1)$$

where M is the range of selection, $C_{(i,j)}$ is the value returned by the individual chromosome i for fitness case j (out of C_t fitness cases) and T_j is the target value for fitness case j. If $|C_{(i,j)} - T_j|$ (the precision) is less than or equal to 0.01, then the precision is equal to zero, and $f_i = f_{max} = C_t \quad M$. In this case, $M = 100$ was used, therefore, $f_{max} = 1000$. The advantage of this kind of fitness functions is that the system can find the optimal solution by itself [26,28]. Then the set of terminals T_{te} and the set of functions F to create the chromosomes are chosen, namely, $T_{te} = \{f_c, b_w, d, a/d, \rho_f, E_f/E_s\}$ and two basic arithmetic operators ($*$, /) and some basic mathematical functions (Cubic Root, Mul3) were used.

Table 3
Range of shear design parameters and V_{cf} for beams used in database.

Input variables	Minimum	Maximum
f_c' (MPa)	24.1	81.4
b_w (mm)	89	1000
d (mm)	141	360
a/d	2.53	6.5
ρ_f	0.0025	0.0302
E_f/E_s	0.16	0.725
Output variable		
V_{cf} (kN)	8.8	190

Another major step is to choose the chromosomal tree, i.e., the length of the head and the number of genes. The GEP approach model initially used single gene and two lengths of heads, and increased the number of genes and heads, one after another during each run, and monitored the training and testing sets performance of the model. In the present study after several trials, length of the head, $h = 6$, and two genes were found to give the best results. The sub-ETs (genes) were linked by multiplication.

Finally, a combination of all genetic operators (mutation, transposition and crossover) is utilized as set of genetic operators. Parameters of the training of the GEP approach model are given in Table 4. Chromosome 32 was observed to be the best of generation individuals predicting V_{cf} having fitness of 837. Explicit formulation based on the GEP approach model for V_{cf} is obtained by

$$V_{cf} = b_w d \left(\sqrt[3]{\frac{d}{a}} f_c' \rho_f \frac{E_f}{E_s} (c_1^2/c_0) \right)^{1/3} (c_0/c_2) \quad (2)$$

The expression tree of formulation for the V_{cf} is also shown also in Fig. 5 where d_0, d_1, d_2, d_3, d_4 and d_5 refer to f_c', b_w, d, a/d, ρ_f and E_f/E_s, respectively. The constants in the formulation are; $c_0 = 7.696$, $c_1 = 7.254$ and $c_2 = 7.718$.

4. Results and discussion

In the present study, all strength reduction factors used in the shear equations for design purposes are set equal to one for comparison. Of the 104 experimental data were utilized for training, testing and validation sets for GEP model. The validation data are unfamiliar to the model and were not included in its development. All of the results obtained from experimental studies and predicted by using the training, testing and validation results of the model are given in Fig. 6. As seen in Fig. 6, the results obtained from the model are compared to the experimental results for training,

Table 4
Parameters of GEP approach model.

Parameter definition	GEP model
p_1 Function set	∗, /, cube root(3Rt), mul3
p_2 Chromosomes	32
p_3 Head size	6
p_4 Number of genes	2
p_5 Linking function	Multiplication
p_6 Mutation rate	0.044
p_7 Inversion rate	0.1
p_8 One-point recombination rate	0.3
p_9 Two-point recombination rate	0.3
p_{10} Gene recombination rate	0.1
p_{11} Gene transposition rate	0.1

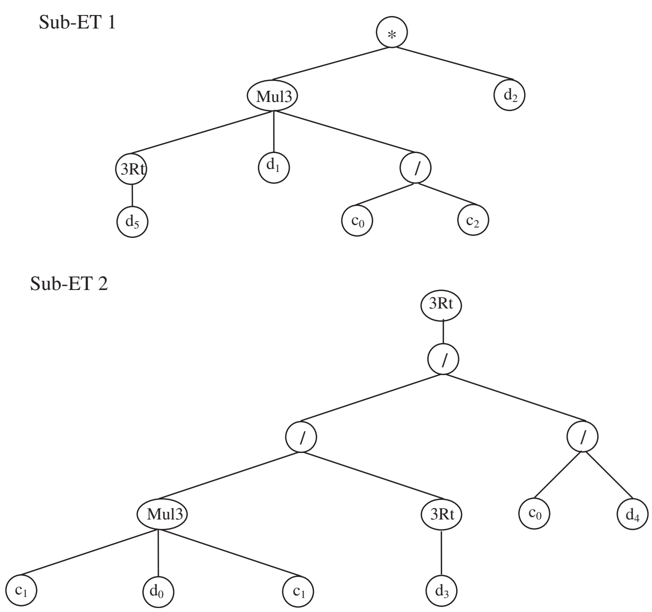


Fig. 5. Expression tree of GEP approach model.

testing and validation sets, respectively. The training set results prove that the proposed model has impressively well learned the non-linear relationship between the input and the output variables with good correlation. Comparing the GEP model predictions with the experimental results for the testing and validation stages demonstrates a high generalization capacity of the proposed model.

The performance of Eq. (2) and equations provided by current shear design guidelines and recommendations in calculating the shear strength of FRP-reinforced concrete beams without stirrups has been evaluated using the training, testing and validation database described earlier, based on both the ratio of experimentally measured to analytically calculated shear strength V_{exp}/V_{cal} (the

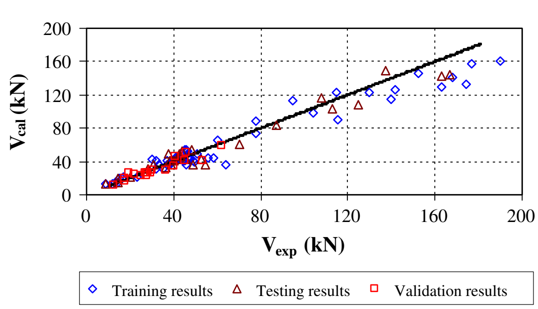


Fig. 6. Comparison of V_{cf} experimental shear strength results with the results of GEP model.

Table 5
Performance of the shear equations considered in this study.

Method	AAE (%)	V_{exp}/V_{cal}	
		Average	SD
ACI 440.1R-03	68.51	3.68	1.45
ACI 440.1R-06	42.3	1.79	0.35
ISIS Canada-01	31.6	1.28	0.37
JSCE-97	22.8	1.32	0.26
CSA S806-02	21.3	1.29	0.21
GEP model	13.4	1.03	0.17

ratio of the shear resistance attained experimentally to the corresponding analytical value), and the average absolute error AAE calculated using Eq. (3).

$$AAE = \frac{1}{n}\sum\frac{|V_{exp} - V_{cal}|}{V_{exp}} \times 100 \tag{3}$$

Table 5 reports the average and standard deviation (SD) for V_{exp}/V_{cal}, and the AAE of all shear design equations. It can be seen that the proposed model has the lowest AAE of 13.4% compared to 42.3% for ACI- 06, 21.3% for CSA-02, 22.8% for JSCE-97, and 31.6% for ISIS-01. The GEP model also provides the least ratio of experimentally measured to analytically calculated shear strength (V_{exp}/V_{cal}) value. Thus, the proposed model appears to be more accurate and reliable for predicting the concrete shear strength for flexural members longitudinally reinforced with FRP bars.

Fig. 7 also shows the performance of the model produced by GEP and those provided by commonly used shear design standards and recommendations. The ratio of experimentally measured to analytically calculated shear strength, V_{exp}/V_{cal} for all beams is shown in the figure. It is clear that the shear design equation provided by the latest version of ACI shear design guidelines for FRP-reinforced beams (ACI 440-06) shows an improved prediction over ACI 440-03, and is beter estimated the shear capacity of FRP-reinforced concrete beams with an average V_{exp}/V_{cal} of 1.79 (3.68 for ACI 440-03). Shear design equations of CSA S806-02, JSCE-97, and ISIS Canada-01 provides better results than that of ACI 440-06. On the other hand, GEP model gives the most accurate results for the shear strength of FRP-reinforced concrete beams with an average V_{exp}/V_{cal} equal to 1.03. The proposed model also includes the most shear design parameters that influence the shear capacity of FRP-reinforced concrete beams as the shear design equation of CSA.

The effect of axial stiffness of FRP bars on the shear capacity FRP-reinforced concrete beams is assumed to be to the magnitude of $(E_f/E_s)^{1/2}$ by ISIS, whereas such an effect is considered to be $(E_f/E_s)^{1/3}$ by Eq. (2), JSE and CSA guidelines. Morever, the ISIS method does not take into account the contribution of other common shear design parameters on V_{cf}, such as the shear span-to-depth ratio, a/d, and the longitudinal reinforcement ratio ρ_f. This could explain why the ISIS method gives the relatively higher value of *SD*.

Fig. 8 through Fig. 10 also present the experimental-to-calculated shear strength versus compressive strength, axial stiffness of reinforcing bars and shear span-to-depth ratio. From the Figs. 8–10, it is evident that the level of accuracy of the shear strength predicted by the GEP model equation seems to be consistent with the varying a/d ratio, compressive strength (f_c') and axial stiffness of reinforcing bars ($\rho_f E_f$).

4.1. Parametric study on effect of basic shear design parameters

4.1.1. Effect of longitudinal reinforcement ratio on shear strength

In the present study, a sensitivity analysis has been conducted using the GEP model to investigate the effect of longitudinal reinforcement ratio on the shear strength of FRP-reinforced concrete

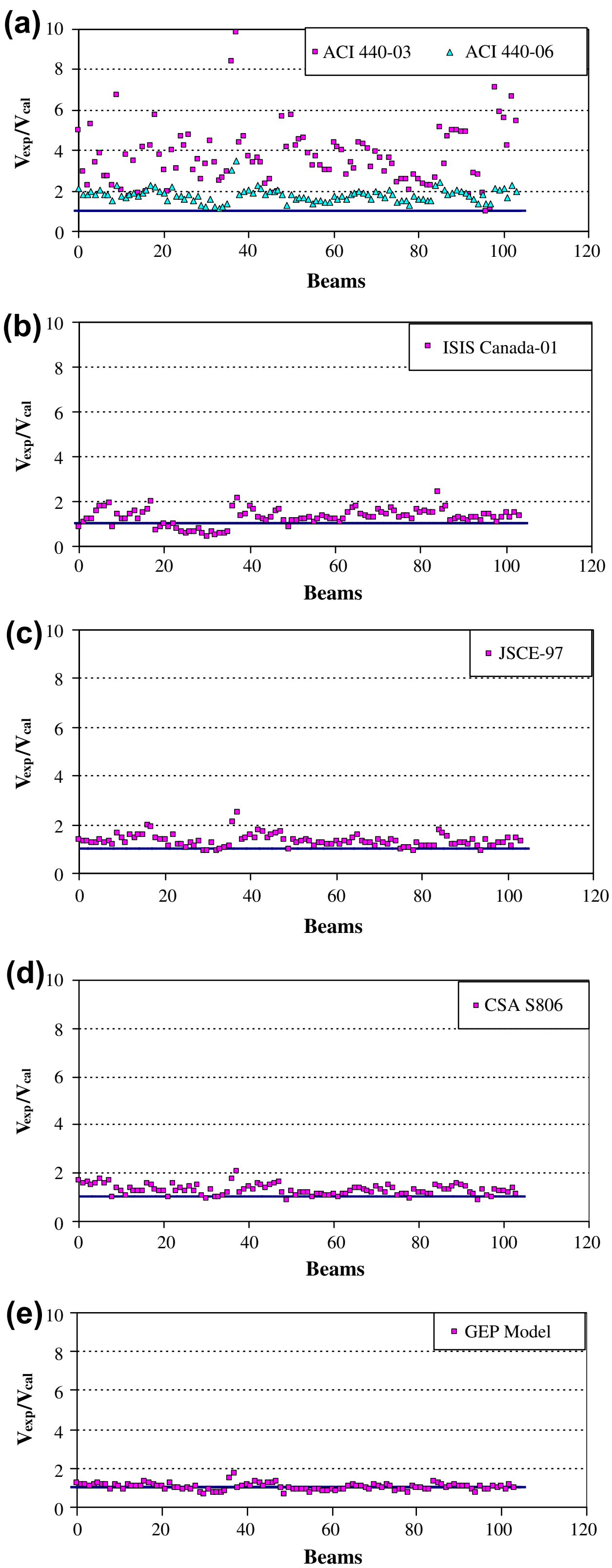


Fig. 7. (a–e) Performance of shear design equations in calculating shear capacity of FRP-reinforced concrete beams without stirrups.

beams without stirrups. The shear strength of a set of beams having geometrical and mechanical properties similar to those of beams randomly selected from the database [8] have been also calculated for different amounts of longitudinal reinforcement ratio using the shear design methods considered herein. Fig. 11 presents the effect of ρ_f on the shear strength of reinforced concrete beams. It is shown that all methods, including the GEP model take into account similar influence for the effect of ρ_f on shear strength. However, a linear relationship is assumed by ACI 440-03 for such an effect, as opposed to a non-linear effect for the other shear design methods.

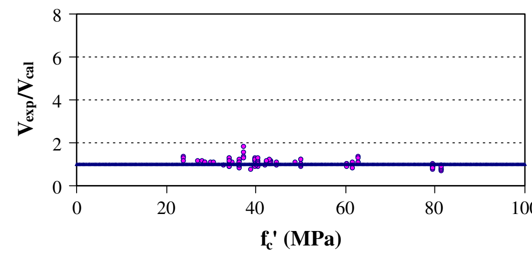


Fig. 8. Experimental to calculated shear strength versus concrete compressive strength.

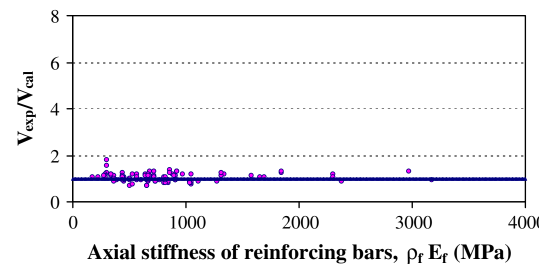


Fig. 9. Experimental to calculated shear strength versus axial stiffness of reinforcing bars.

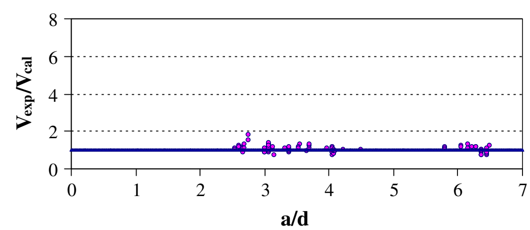


Fig. 10. Experimental to calculated shear strength versus shear span-to-depth ratio.

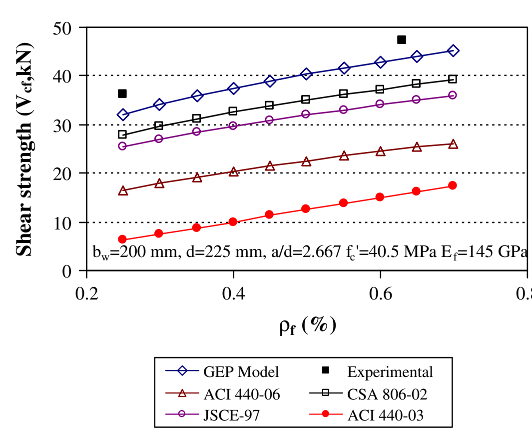


Fig. 11. Effect of the longitudinal reinforcement ratio (ρ_f) on the shear strength of FRP-reinforced concrete beams without stirrups.

4.2. Effect of concrete compressive strength on shear strength

To investigate the ability of shear design guidelines to quantitatively consider the effect of f_c' on shear strength of FRP-reinforced concrete beams, a set of six beams similar to a beam randomly selected from the database and tested by Razaqpur et al. [8] is considered. Fig. 12 shows the variation in shear strength of FRP-reinforced concrete beams with variable concrete compressive strength. The figure illustrates the effect of f_c' as estimated by the GEP model and various shear provisions considered in this study. It is shown that all shear design methods consider the effect of f_c', but they vary in the magnitude of such an effect. The shear design method provided by ACI 440-03 assumes that the shear strength of FRP-reinforced concrete beams decreases as f_c' increases, whereas all other methods, including the GEP model assume that the shear strength of FRP-reinforced concrete beams increases with an increase of concrete compressive strength (Fig. 12).

4.3. Effect of shear span-to-depth ratio on shear strength

Fig. 13 shows the relationship between shear span-to-depth ratio (a/d) and the shear strength of a set of beams calculated using the GEP model and shear design methods considered herein. The figure also includes the experimental shear strength of a similar beam measured by El-Sayed et al. [30]. While ACI 440 and JSCE shear design provisions do not consider the effect of a/d on the shear strength of reinforced concrete beams, CSA S806 and GEP model responses exhibit a slight influence of a/d on the shear strength of FRP-reinforced concrete slender beams ($a/d > 2.5$). However, previous studies [41,42] indicated that smaller values of a/d have a larger effect on the shear capacity of FRP-reinforced concrete short beams ($a/d < 2.5$). Due to lack of suffficient experimental data on FRP-reinforced concrete short beams, GEP model has been developed for the shear strength of FRP-reinforced concrete slender beams ($a/d > 2.5$) in this study.

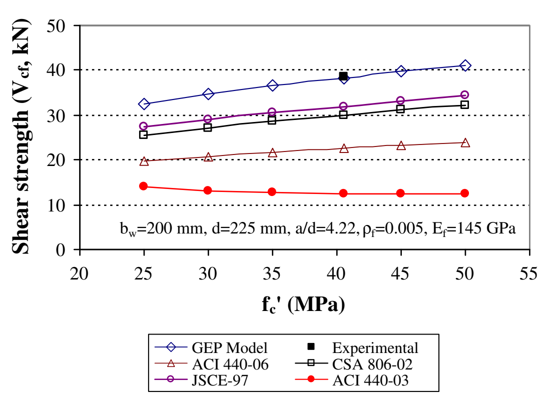


Fig. 12. Effect of the concrete compressive strength (f_c') on the shear strength of FRP-reinforced concrete beams without stirrups.

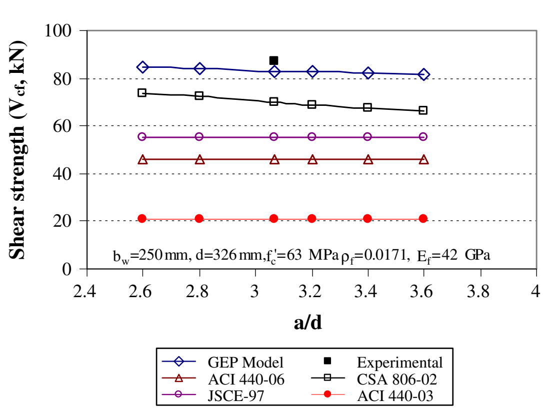


Fig. 13. Effect of the shear span-to-depth ratio (a/d) on the shear strength of FRP-reinforced concrete beams without stirrups.

5. Conclusions

This study reports an efficient approach for the formulation of shear strength of FRP-reinforced concrete beams using GEP. An empirical model to predict the shear strength of FRP-reinforced concrete beams without web reinfocement has been obtained by GEP approach. Experimental results are used to build and validate the model. Good agreement between the model predictions and experiments has been achieved. The values of the average absolute error AAE, and the average and standard deviation for V_{exp}/V_{cal} have shown this situation.

The GEP model equation also gives good predictions for the shear strength of FRP-reinforced concrete beams with the varying a/d ratio, compressive strength of concrete and axial stiffness of reinforcing bars.

Shear provisions of ACI 440 are highly conservative in estimating the shear strength of FRP-RC beams without shear reinforcement. All other shear provisions considered in this study also gives conservative results for such beams even without applying reduction factors.

The proposed model has been compared to the current guidelines and provisions. More accurate and consistent predictions have been obtained using the model produced by GEP.

Shear design equation produced by GEP model accounts for the effect of the axial stiffness of FRP bars on shear capacity of FRP-reinforced concrete beams as a cubic root function of E_f/E_s. It provides the most accurate results in calculating V_{cf}.

The proposed model is so simple that they can be used by anyone not necessarily being familiar with GEP. The model also gives a practical way for the prediction of concrete shear strength of beams reinforced with FRP bars to obtain accurate results, and encourages use of GEP in other aspects of civil engineering studies.

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